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SOFIA Primary Mirror Dynamic Distortion and Dynamic Displacement

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System Engineering Report

SOFIA PRIMARY MIRROR DYNAMIC DISTORTION AND DYNAMIC DISPLACEMENT

ABSTRACT

A preliminary dynamic analysis of the SOFIA primary mirror has been performed. Two quantities were sought:

1) the surface distortion of the primary mirror caused by vibratory inputs (distortion)
2) the translational motion of the primary mirror relative to the secondary mirror along the optic axis of the telescope (axial displacement or despace).

The analysis was based upon a direct frequency analysis of the telescope structure and the equation for vibratory magnification,

$$H(\omega) = \{[1 - (\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2\}^{-1/2} \quad (1)$$

where $H(\omega)$ is the magnification factor.Results give distortion and axial displacement values for three frequencies - 25,104 and 311 Hz (the fundamental frequencies of the telescope structure, the primary mirror support frame and the primary mirror, respectively).

Worst case mirror distortion is 5.155 nm at 25 Hz and this is principally due to the resonating of the telescope structure. Worst case axial displacement is 2.058 nm at 104 Hz and is the result of the primary mirror support frame resonating causing an axial motion relative to the secondary mirror. Both of these values are small relative to the 0.21 μm rms mirror distortion budget and the 5 μm despace budget.

INTRODUCTION

SOFIA primary mirror distortions due to dynamic effects are caused by vibration transmitted from the aircraft, through a vibration isolation system, through an air bearing, to the telescope structure, through the primary mirror support frame, and to the mirror itself. Discussion of vibration transmitted from the aircraft and isolated before reaching the air bearing is not treated in this report (Ref. "Vibration Isolation System," T. M. Kaiser). This SER concerns itself with the effect on the primary mirror surface and its motion relative to the secondary mirror due to vibratory inputs at the air bearing. The values obtained are approximate as simplifying assumptions have been made, however, errors associated with these results should not be of such magnitude that general conclusions are misleading.

ANALYSIS

Because the telescope is cantilevered from the air bearing, a vibratory input is amplified through the length of the telescope's support arm. A NASTRAN direct frequency analysis has been performed on the telescope structure and a fundamental frequency of 25 Hz used (additional work by B. Banfield has predicted slightly different natural frequencies). From this analysis, structural amplification factors for various frequencies have been determined relating output amplitudes at the telescope structure to input amplitudes at the air bearing. Table 1 is a summary of the direct frequency analysis for acceleration along the optic axis of the telescope. The first column indicates the frequencies within the analysis which were investigated. The second column gives a ratio of the output displacement at the telescope structure to the input displacement at the air bearing - the structural amplification factors. Column three indicates input acceleration and column four, the output acceleration. The final column is the response displacement at the telescope structure due to the input acceleration at the air bearing.

The direct frequency analysis modeled the primary mirror support frame as rigid with a lumped mirror mass rigidly attached at its center. Since the direct frequency analysis was performed, detailed models of the support frame and primary mirror have been run independently. The following discussion attempts to infer worst case vibratory conditions incorporating the telescope structure, the support frame and the primary mirror models. This analysis assumed the primary mirror support system (i.e., mounts) to be a system of relatively stiff links. Their actual elastic influence on these results was not estimated. Also, the support frame and primary mirror were modeled as single degree of freedom spring-damper-mass systems. The results of their individual analyses were superimposed to obtain the values presented in this report.

From independent modal analyses, fundamental frequencies for the support frame and primary mirror were found to be 104 Hz and 311 Hz, respectively. Assuming structural damping to be 1% of critical (i.e., $\zeta = 0.01$) and amplitude magnification given by Equation 1 (depicted in Figure 1), consider the resonant condition of each of the three telescope system components - the telescope structure, the primary mirror support frame and the primary mirror.

Table 1. Results of Telescope Structure Direct Frequency Analysis

Frequency, Hz	Structural Amplification Factor	Input Acceleration, μg	Output Acceleration, μg	Response Displacement, nm
25	2.7	4.50	12.15	4.826
100	1.5	1.12	1.68	0.042
200	2.6	0.56	1.46	0.009
300	1.1	0.37	0.41	0.001
400	1.2	0.28	0.34	0.001

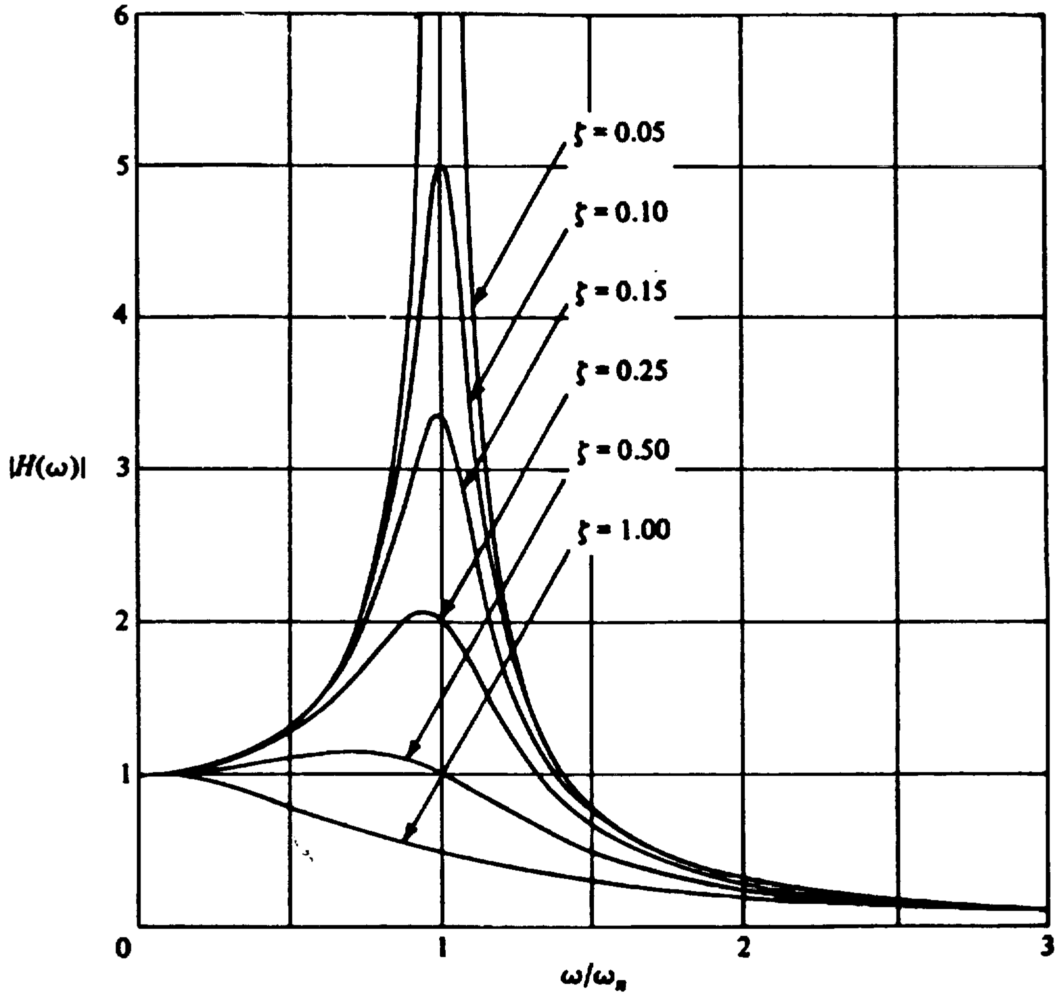

Figure 1. Absolute Value of Magnification Factor vs. Frequency Ratio

$$H(\omega) = \{[1 - (\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2\}^{-1/2}$$

RESULTS

The analysis was done similarly in each of the three cases described below. For a given frequency, the dynamic amplitude of the telescope structure was taken from the results of the direct frequency analysis. The magnification factor for the support frame was determined and multiplied to this telescope structure amplitude. The result was the displacement at the support frame due to input at the air bearing. The magnification factor for the primary mirror was then determined and multiplied to this displacement at the support frame. The result was the total displacement of the primary mirror surface due to vibratory input at the air bearing. The change in distance of the primary mirror relative to the secondary mirror (despace) was taken as the amplitude of the support frame vibration (the difference between the first and second column, Table 2).

At 25 Hz, the telescope structure was at resonance and, as predicted by the direct frequency analysis, displaced 2.7 times the input vibration at the air bearing, or 4.826 nm. From Equation 1, the support frame's amplitude magnification factor was calculated at 1.603 (see Appendix for calculations). The amplitude at the support frame was found by multiplying the telescope structure amplitude by this magnification factor. The maximum dynamic displacement at the support frame was thus 5.122 nm. For the primary mirror at 25 Hz, Equation 1 gave a magnification factor of 1.007 and the amplitude at the primary mirror surface was found to be 5.155 nm. This value was taken to be the surface distortion to the primary mirror due to vibratory inputs. The motion along the telescope's optic axis of the primary mirror relative to the secondary mirror was determined by subtracting the telescope structure amplitude from the support frame amplitude; 0.296 nm.

The first resonant condition of the support frame was given at 104 Hz. The telescope structure had already passed through its fundamental frequency and, from the results of the direct frequency analysis, a structural amplification factor (100 Hz) of 1.5 yielded a response displacement of 0.042 nm. From Equation 1, the magnification factor for support frame was calculated to be 50. Thus a 0.042 nm displacement at the telescope structure was amplified to a value of 2.100 nm. Again using Equation 1, the primary mirror's amplitude magnification factor was calculated at 1.123. In total, vibration at 104 Hz saw a 0.042 nm contribution from the telescope structure, a magnification by the support frame to 2.100 nm and another magnification by the primary mirror to 2.358 nm. At this frequency, the resonance of the support frame caused a despace of 2.058 nm. This value is more than three orders of magnitude below the allowable 5 μm despace stability tolerance.

The first primary mirror resonant condition was at 311 Hz. The direct frequency analysis of the telescope structure gave a structural amplification factor (300 Hz) of 1.1 and a response displacement of 0.001 nm. Calculating an amplitude magnification factor of 0.1259 for the support frame, the dynamic displacement was decreased 0.0001 nm. The primary mirror was at resonance and thereby its magnification factor was equal to 50. This amplification brought the total dynamic displacement at 311 Hz to 0.005 nm. Considering vibratory modes of

Table 2. Dynamic Displacements of SOFIA Telescope System

Frequency, Hz	Telescope Structure, nm	Support Frame, nm	Primary Mirror (Distortion), nm	Primary Mirror (Despace), nm
25	4.826	5.122	5.155	0.296
104	0.042	2.100	2.358	2.058
300	0.001	0.0001	0.006	0.0009

the primary mirror at resonance, it is possible to have the center of gravity of the mirror displacing 0.005 nm in one direction while another point on the mirror moves 0.005 nm in the other direction. Vibration in this mode could contribute 0.010 nm to the primary mirror surface distortion. Despace at this frequency was 0.0009 nm.

CONCLUSIONS

Table 2 summarizes dynamic displacements of the three SOFIA primary mirror components considered in this SER. Worst case primary mirror surface distortion, 5.155 nm, occurred at 25 Hz. This value is well below the 0.21 μm rms primary mirror distortion budget allocated by the Optics Group. Worst case primary mirror despace was 2.058 nm and well below the Optics Group budget.

The above values are approximate. The purpose of this SER was to make use of available data and predict order of magnitude results. This analysis indicates that dynamic distortion and displacements are small relative to their allocated budgets. Future work is to include a comprehensive model of the entire telescope - telescope structure, support frame, support system and primary mirror.

APPENDIX

SOFIA PRIMARY MIRROR DYNAMIC DISTORTION & TRANSLATION

FROM BRANFELD'S [illegible] ANALYSIS (SEE ATTACHED SHEET)

FREQUENCY, Hz	AMPLITUDE OF TELESCOPE STRUCTURE, nm
25	4.326
100	0.042
300	0.001

AMPLITUDE MAGNIFICATION FACTOR, $H(\omega)$

$$H(\omega) = \frac{1}{\left\{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left[2\zeta\left(\omega/\omega_n\right)\right]^2\right\}^{1/2}}$$

① CONSIDER VIBRATION OF TELESCOPE @ 25 Hz (ASSUME $\zeta = 0.01$)

AMPLITUDE OF TELESCOPE STRUCTURE: 4.826 nm

Re SUPPORT FRAME ($f_n = 104$ Hz)

$$H(25) = \frac{1}{\left\{\left[1-\left(25/104\right)^2\right]^2 + \left[2(0.01)\left(25/104\right)\right]^2\right\}^{1/2}}$$

$= 1.0613$

AMPLITUDE AT SUPPORT FRAME = (4.326 nm)(1.0613) = 5.122 nm

Re PRIMARY MIRROR ($f_n = 311$ Hz)

$$H(25) = \frac{1}{\left\{\left[1-\left(25/311\right)^2\right]^2 + \left[2(0.01)\left(25/311\right)\right]^2\right\}^{1/2}}$$

$= 1.007$

AMPLITUDE AT PRIMARY MIRROR = (5.122 nm)(1.007) = 5.155 nm

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AMPLITUDE OF MOTION OF PRIMARY MIRROR RELATIVE TO SECONDARY MIRROR (i.e. DESPACE)

$$5.122\,nm - 4.826\,nm = \underline{\underline{0.296\,nm}}$$

(2) CONSIDERING VIBRATION OF TELESCOPE @ 104 Hz ($\zeta = 0.01$)

AMPLITUDE OF TELESCOPE STRUCTURE: $\underline{\underline{0.042\,nm}}$

Re SUPPORT FRAME

$$H'(104) = \frac{1}{2\zeta} = \frac{1}{2(0.01)} = 50$$

AMPLITUDE AT SUPPORT FRAME $= (0.042\,nm)(50) = \underline{\underline{2.100\,nm}}$

Re PRIMARY MIRROR

$$H(104) = \frac{1}{\left\{\left[1-\left(^{104}/_{311}\right)^2\right]^2 + \left[2(0.01)\left(^{104}/_{311}\right)\right]^2\right\}^{1/2}}$$

$= 1.123$

AMPLITUDE AT PRIMARY MIRROR $= (2.100\,nm)(1.123) = \underline{\underline{2.358\,nm}}$

DESPACE OF PRIMARY MIRROR $= 2.100\,nm - 0.042\,nm = \underline{\underline{2.058\,nm}}$

(3) CONSIDER VIBRATION OF TELESCOPE @ 311 Hz ($\zeta = 0.01$)

AMPLITUDE OF TELESCOPE STRUCTURE: $\underline{\underline{0.001\,nm}}$

Re SUPPORT FRAME

$$H(311) = \frac{1}{\left\{\left[1-\left(^{311}/_{104}\right)^2\right]^2 + \left[2(0.01)\left(^{311}/_{104}\right)\right]^2\right\}^{1/2}}$$

$= 0.126$

AMPLITUDE AT SUPPORT FRAME $= (0.001\,nm)(0.126) = \underline{\underline{0.00013\,nm}}$

Re PRIMARY MIRROR

$$L(311) = \frac{1}{2\zeta} = \frac{1}{2(0.01)} = 50$$

AMPLITUDE AT PRIMARY MIRROR = $(0.00013\,nm)(50) = \underline{\underline{0.0063\,nm}}$

DISPLACE OF PRIMARY MIRROR = $|0.00013\,nm - 0.001\,nm| = \underline{\underline{0.0009\,nm}}$

SOFIA

DIREC F. 5D ANALYSIS

		RESPONSES T_1 (Lateral)	T_2 (Lateral)	T_3 (Axial)
N2536 (Prim. Mirror)	(1) 25 Hz	6.302×10^{-4}	3.026×10^{-4}	3.145×10^{-4}
	(2) 100 Hz	9.993×10^{-5} *	1.893×10^{-5}	6.955×10^{-5} * ✓
	(3) 200 Hz	5.625×10^{-5} *	3.452×10^{-4}	2.334×10^{-4}
	(4) 300 Hz	1.525×10^{-6}	6.118×10^{-6} *	1.766×10^{-6} * ✓
	(5) 400 Hz	1.221×10^{-5}	7.839×10^{-6}	1.959×10^{-6} *
N2537 (Air Bearing)	(1) 25 Hz	1.447×10^{-4}	9.402×10^{-5}	1.174×10^{-4}
	(2) 100 Hz	1.794×10^{-4}	2.354×10^{-5}	1.332×10^{-4} ✓
	(3) 200 Hz	2.876×10^{-4}	4.419×10^{-5}	8.945×10^{-5}
	(4) 300 Hz	1.070×10^{-5}	2.191×10^{-5}	1.958×10^{-5} ✓
	(5) 400 Hz	1.124×10^{-5}	9.308×10^{-6}	8.247×10^{-6} ✓
Ratios $\left(\frac{P.M.}{A.B.}\right)$	(1)	4.4	3.2	2.7
	(2)	1.6	0.8	1.5
	(3)	1.2	7.8	2.6
	(4)	0.1	1.3	1.1
	(5)	1.1	0.8	1.2

	INPUT $\mu G^2/Hz$	FORCING FREQ. Hz	TRANSMITTED ACCEL. AT A/B μG	TRANSMITTED ACCEL AT P.M. μG	DISPL. RESPONSES AT P.M. (TELESCOPE STRUCT) nm ($\times10^{-9}$m)
VERT. Accel.	1000	25	4.50	2.7 × 4.50 = 12.15	4.826
		100	1.12	1.5 × 1.12 = 1.68	0.042
		200	0.56	2.6 × 0.56 = 1.46	0.009
		300	0.37	1.1 × 0.37 = 0.41	0.001
		400	0.28	1.2 × 0.28 = 0.34	0.001
LAT. Accel.	525	25			
		100			
		200			
		300			
		400			

$x = A\sin\omega t$

$\ddot{x} = A\omega^2 \sin\omega t$

$a_{max} = A\omega^2$

N2537